



SE-6149

B. E. II (Sem. III) (Civil) Examination

April / May – 2011

Engineering Mathematics & Statistical Methods

Time : 3 Hours]

[Total Marks : 100

Instructions :

(1)

नीचे दशांशों में निशानीवाणी विगतो उत्तरवली पर अवश्य कभवी. Fillup strictly the details of signs on your answer book.	Seat No. :
Name of the Examination :	<input type="text"/>
<input type="text" value="B. E. 2 (Sem. 3) (Civil)"/>	<input type="text"/>
Name of the Subject :	<input type="text"/>
<input type="text" value="Engg. Mathematics & Statistical Methods"/>	<input type="text"/>
Subject Code No. : <input type="text" value="6"/> <input type="text" value="1"/> <input type="text" value="4"/> <input type="text" value="9"/>	<input type="text"/>
Section No. (1, 2,.....) : <input type="text" value="Nil"/>	<input type="text"/>
	Student's Signature

- (2) All questions are compulsory.
(3) Figures to the right indicate full marks of the questions.

1 (a) Do as directed :

(1) Evaluate $\int_0^1 \int_0^x e^{y/x} dy dx$ 3

(2) Prove that $\nabla(r^n) = nr^{n-2} \bar{r}$ 3

(3) Evaluate $\int_a^b (x-a)^m (b-x)^n dx$ and 4

hence find the value of $\int_3^7 \sqrt[4]{(x-3)(7-x)} dx$

(b) Attempt any four : 16

(1) Evaluate $\iint_R r \sin \theta dr d\theta$ over the area of the cardioid $r = a(1 + \cos \theta)$ above the initial line.

(2) Evaluate $\int_0^1 \int_{4y}^4 e^{x^2} dA$ by changing the order.

- (3) Evaluate $\int_0^{\log 2} \int_0^x \int_0^y e^{x+y+z} dz dy dx$
- (4) Find the area bounded by the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ in the first quadrant.
- (5) Find the volume bounded by xy-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$

2 (a) Attempt any two : 6

- (1) Find the directional derivative of $\phi = 3e^{2x-y+z}$ at A (1, 1, -1) in the direction \overrightarrow{AB} , where B is the point (-3, 5, 6).
- (2) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$
- (3) Show that $\overline{F} = 2xyz i + (x^2 z + 2y) j + x^2 y k$ is irrotational and find a scalar function ϕ such that $\overline{F} = \text{grad } \phi$

(b) Attempt any two : 8

- (1) Using Green's theorem evaluate $\oint_c [(xy - x^2) dx + x^2 y dy]$ along the closed curve C formed by $y = 0$, $x = 1$ and $y = x$.
- (2) Use divergence theorem to evaluate $\iiint_S \overline{F} \cdot d\overline{s}$ where $\overline{F} = x^3 i + y^3 j + z^3 k$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$
- (3) Evaluate $\oint_c [(2x - y) dx - yz^2 dy - y^2 z dz]$ using Stoke's theorem, where C is the boundary $x^2 + y^2 = 1$

3 Attempt any two : 10

- (1) Evaluate $\int_0^1 x^m (\log x)^n dx$, where n is a +ve integer and

$m > 1$ hence find $\int_0^1 x^4 (\log x)^3 dx$

(2) Evaluate $\int_0^{\infty} \frac{x^c}{e^x} dx$, hence find $\int_0^{\infty} \frac{x^4}{4^x} dx$

(3) Prove that $\int_0^{\infty} e^{-x^2-2bx} dx = \frac{\sqrt{\pi}}{2} e^{b^2} (1 - \text{erf}(b))$

- 4 (a) Answer the following questions :
- (1) Solve $yp - xq = 0$ 3
 - (2) Write an IBVP for the 3
 - (i) Wave eqⁿ. and (ii) Heat eqⁿ.
 - (3) The overall percentage of failure in certain examination is 20 %. If six candidates in the exam appear, what is the probability that at least five pass the exam ? 4
- (b) Find the deflection $u(x,t)$ of the vibrating string of length π , and ends are fixed, corresponding to zero initial velocity and initial deflection, $f(x) = k(\sin x - \sin 2x)$, given $c^2 = 1$ 8

OR

- (b) An insulated rod of length l has its ends A and B kept at $a^\circ\text{C}$ and $b^\circ\text{C}$ respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to 0°C and kept so. Find the temperature distribution at any point on rod from the end A. 8
- 5 (a) Attempt any two : 05
- (1) Solve : $pz - qz = z^2 + (x+y)^2$
 - (2) Solve : $(2z - y)p + (x+z)q + 2x + y = 0$
 - (3) Solve : $x^2p + y^2q = (x+y)z$
- (b) Attempt any two : 10
- (1) Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six?
 - (2) A car-hire firm has two cars, which it hires out day by day. The number of demand for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused.

$$(e^{-1.5} = 0.2231)$$

- (3) A sample of 100 dry battery cells tested to find the length of life produced the following results :
 $\bar{x} = 12$ hours and $\sigma = 3$ hours.
 Assuming the data to be normally distributed, what percentage of battery cells are expected to have life,
 (i) more than 15 hours, (ii) less than 6 hours.
 (The S.N.V.Z. area between 0 and 1 is 0.3413 and between 0 and 2 is 0.4772)

- 6 (a) Define statistic χ^2 . Write the condition for 5
 application of χ^2 test and write the application of χ^2 test.
 (b) Attempt any two : 12
 (1) Ten students got the following percentage of marks in Principles of Economics and Statistics :

<i>Roll Nos.</i>	1	2	3	4	5	6	7	8	9	10
<i>Marks in Economics</i>	78	36	98	25	75	82	90	62	65	39
<i>Marks in Statistics</i>	84	51	91	60	68	62	86	58	53	47

Calculate the coefficient of correlation.

- (2) Fit a straight line for the following data :

<i>x</i>	50	70	100	120
<i>y</i>	12	15	21	25

- (3) Calculate the coefficient of correlation and obtain the least square regression line of y on x has the following data :

x : 1 2 3 4 5 6 7 8 9
 y : 9 8 10 12 11 13 14 16 15